



Hybrid Hydro+Micro Models: Status and Outlook

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- 3+1D ideal RFD
- hybrid Hydro+Micro Models
- η/s of a Hadron Gas

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C. Nonaka

work supported through grants by





The Case for Hydro & Hydro+Micro



RHIC in the press: Perfect Liquid

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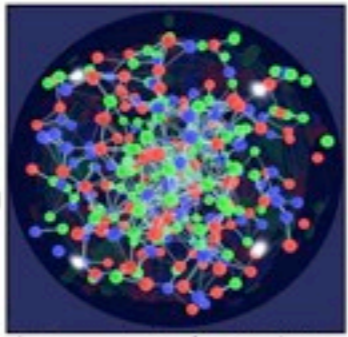
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Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.



The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

The work is expected to help scientists explain the conditions that existed just milliseconds after the Big Bang.

The details, presented to the American Physical Society in Florida, will be published across a number of papers in the journal Nuclear Physics A.

They summarise the work of four collaborative experiments - dubbed Brahms, Phenix, Phobos and Star - which have been running on Brookhaven's Relativistic Heavy Ion Collider (RHIC).

First moments

Already, the results have caused quite a stir in the research community.

"The experimental collaborations are still taking a cautious approach whereas people like me, who use model calculations, are already so excited about the data because we believe they have actually found the elusive state known as the quark-gluon plasma," commented theoretical nuclear physicist Steffen Bass from Duke University.

The QGP is the state postulated to be present just a few

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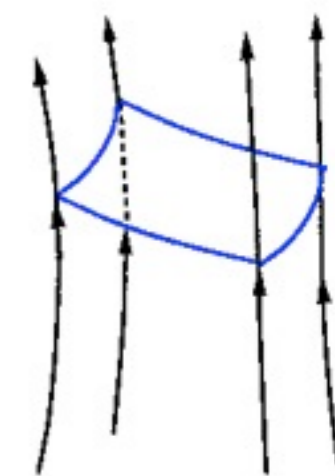
- on April 18th, 2005, BNL announced in a press release that RHIC had created a new state of hot and dense matter which behaves like a nearly perfect liquid.
- how does one measure/calculate the properties of an ideal liquid?
- are there any other ideal liquid systems found in nature?

Relativistic Fluid Dynamics

- transport of macroscopic degrees of freedom
- based on conservation laws: $\partial_\mu T^{\mu\nu}=0$ $\partial_\mu j^\mu=0$
- for ideal fluid: $T^{\mu\nu} = (\varepsilon+p) u^\mu u^\nu - p g^{\mu\nu}$ and $j_i^\mu = \rho_i u^\mu$
- **Equation of State** needed to close system of PDE's: $p=p(T,\rho_i)$
- connection to Lattice QCD calculation of EoS
- initial conditions (i.e. thermalized QGP) required for calculation
- Hydro assumes local thermal equilibrium, vanishing mean free path

This particular implementation:

- fully 3+1 dimensional, using (τ,x,y,η) coordinates
- Lagrangian Hydrodynamics
 - coordinates move with entropy-density & baryon-number currents
 - trace adiabatic path of each volume element



3D-Hydro: Parameters

Initial Conditions:

- Energy Density:

$$\epsilon(x, y, \eta) = \epsilon_{\max} W(x, y; b) H(\eta)$$

- Baryon Number Density:

$$n_B(x, y, \eta) = n_{B\max} W(x, y; b) H(\eta)$$

Parameters:

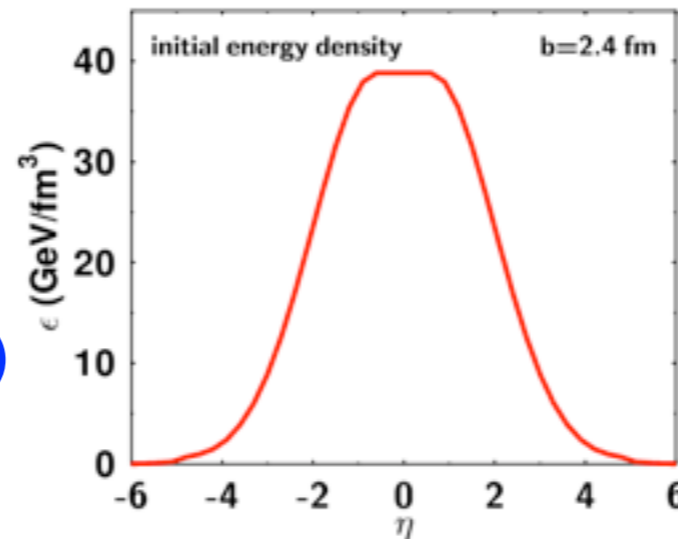
$$\begin{cases} \tau_0 = 0.6 \text{ fm/c} \\ \epsilon_{\max} = 55 \text{ GeV/fm}^3, n_{B\max} = 0.15 \text{ fm}^{-3} \\ \eta_0 = 0.5 \quad \sigma_\eta = 1.5 \end{cases}$$

- Initial Flow: $v_L = \eta$ (Bjorken's solution); $v_T = 0$

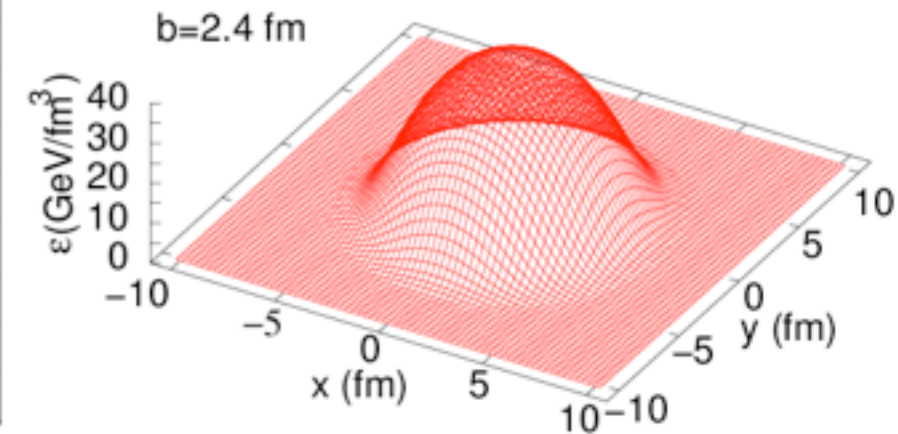
Equation of State:

- Bag Model + excluded volume
- 1st order phase transition (to be replaced by Lattice EoS)

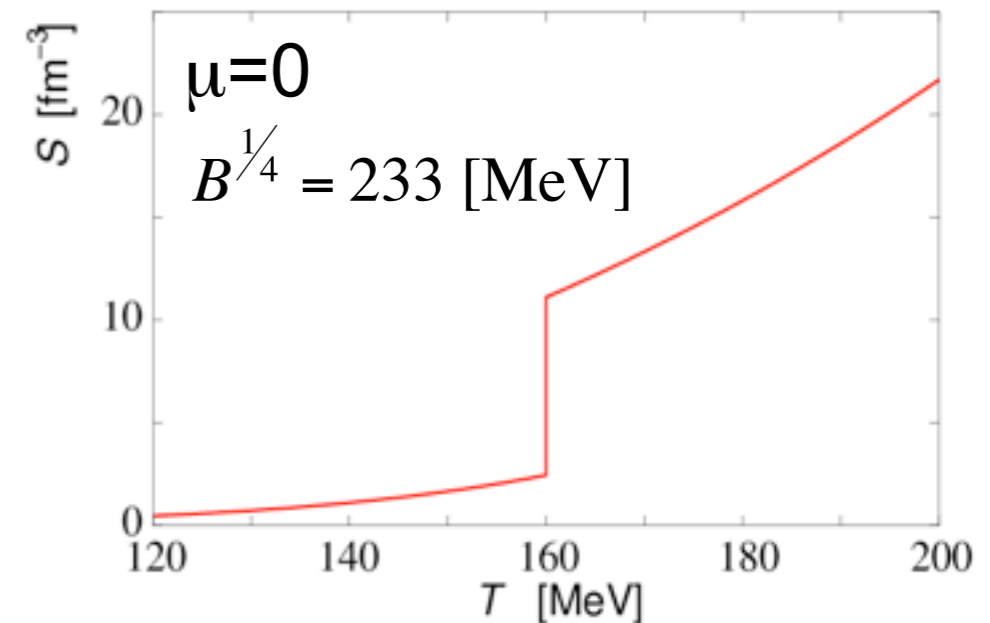
longitudinal profile:



transverse profile:

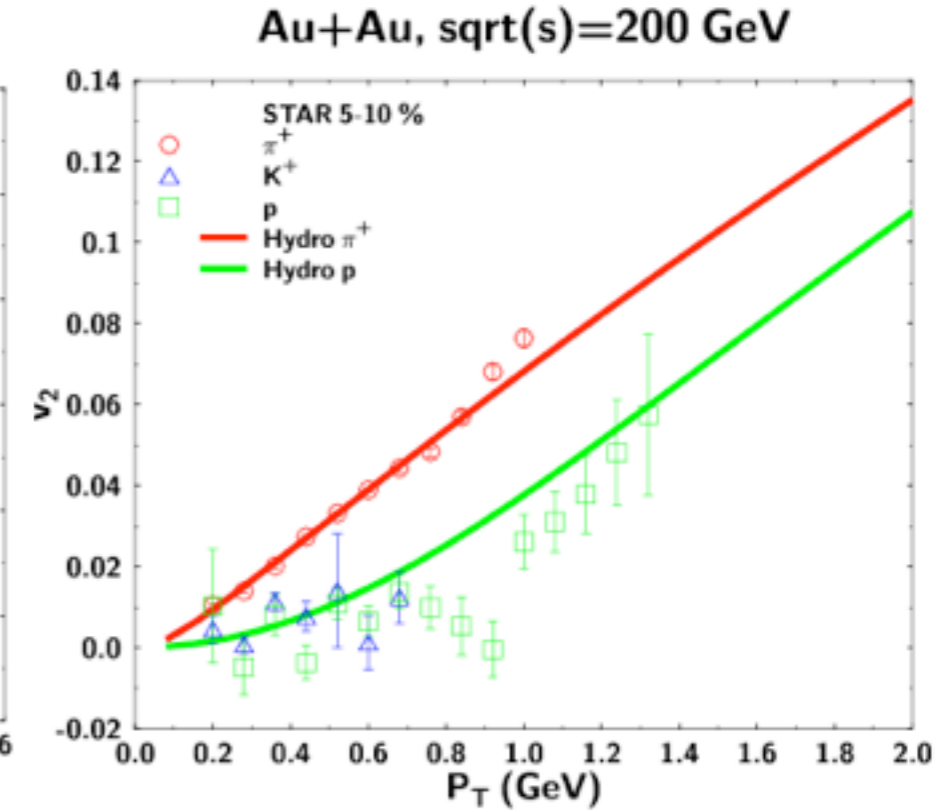
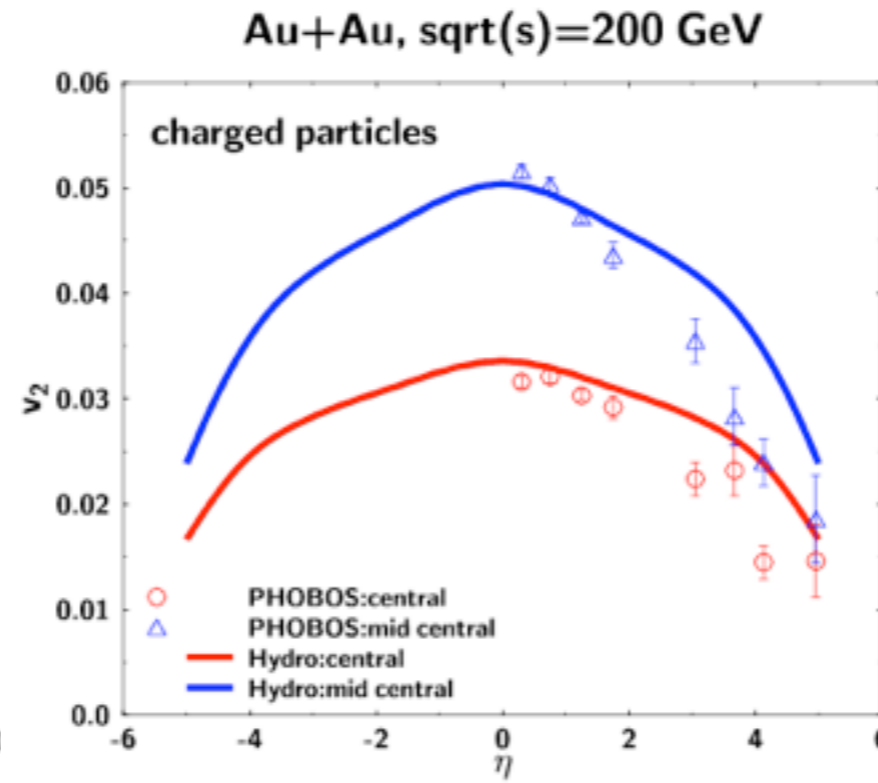
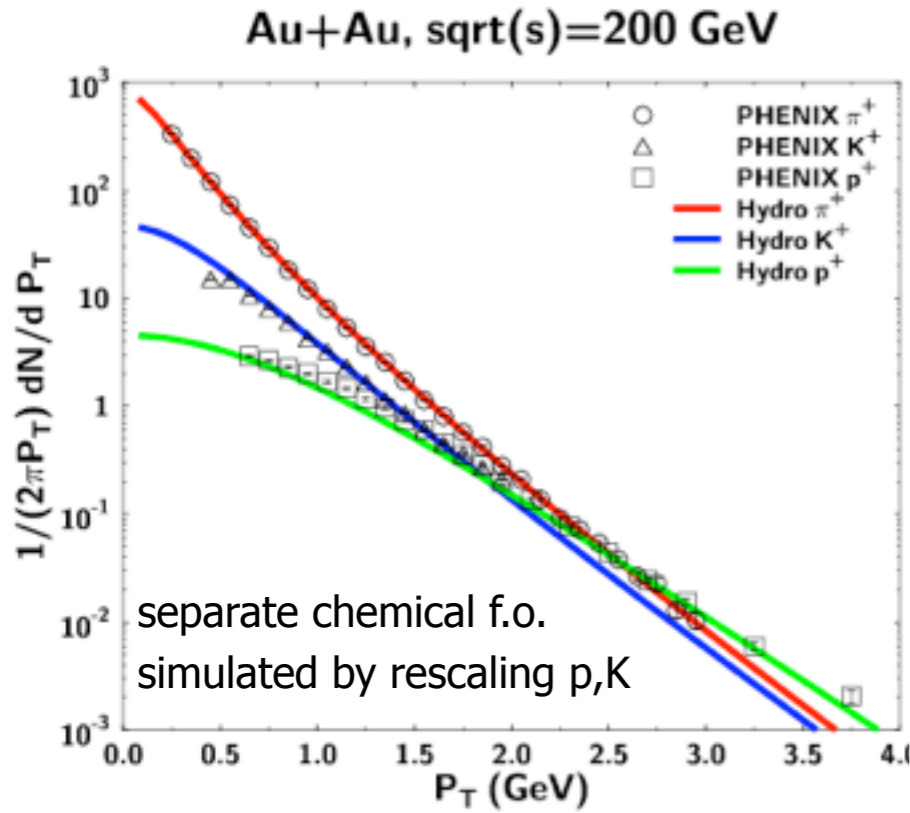


EOS (entropy density)

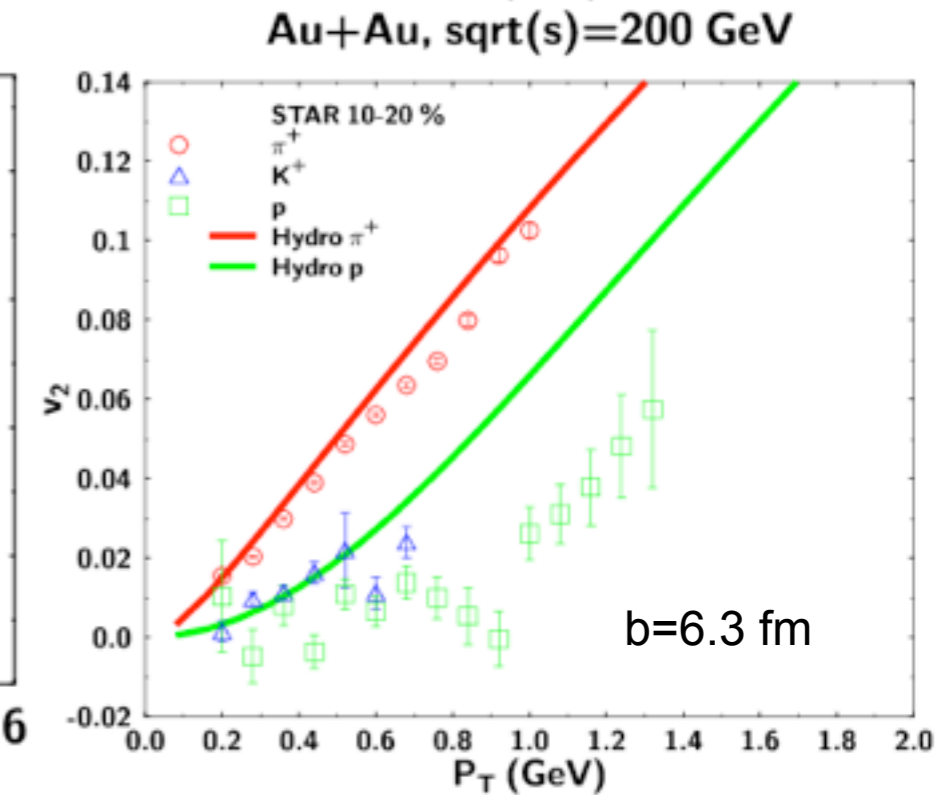
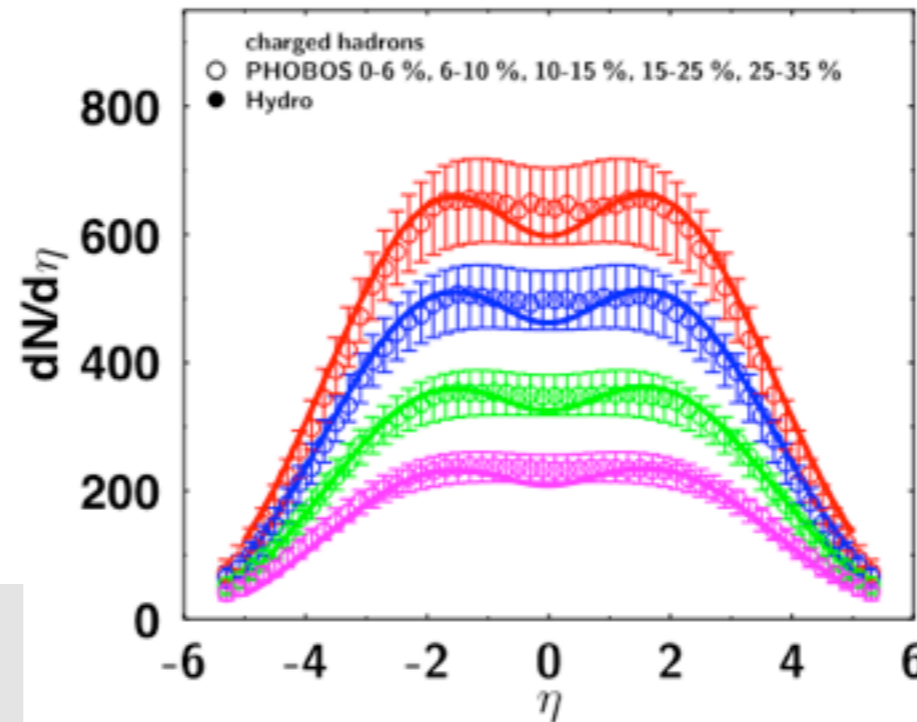




3D-Hydro: Comparison to RHIC



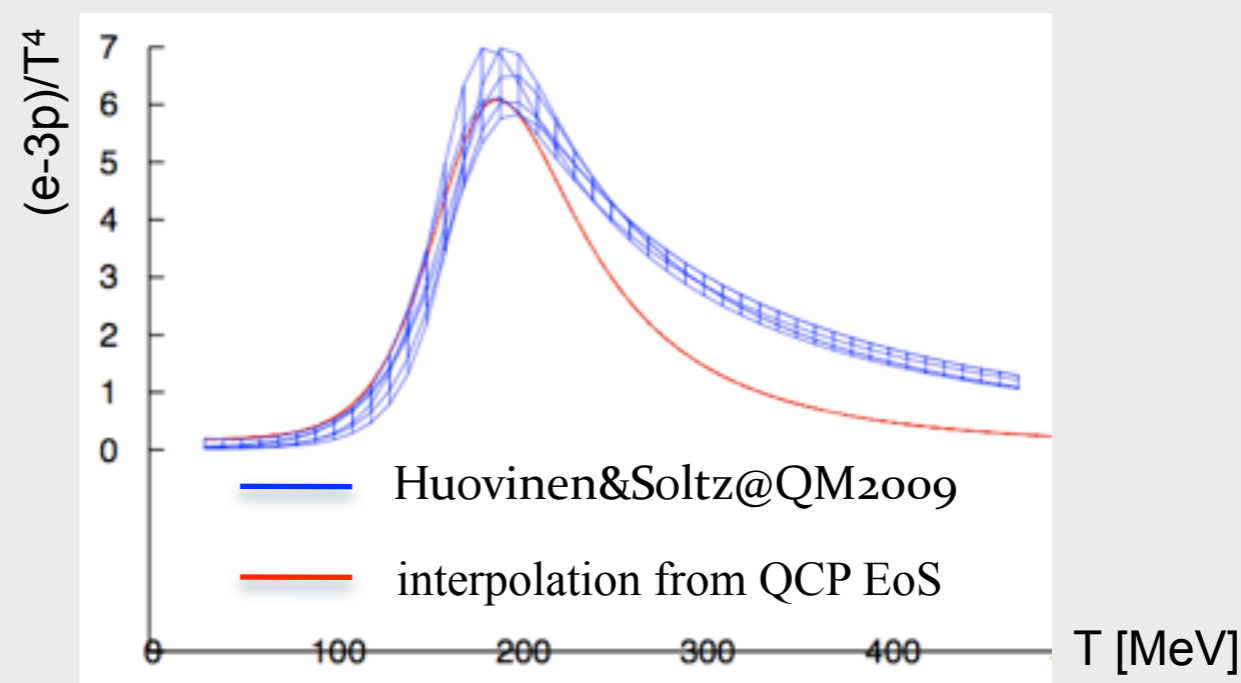
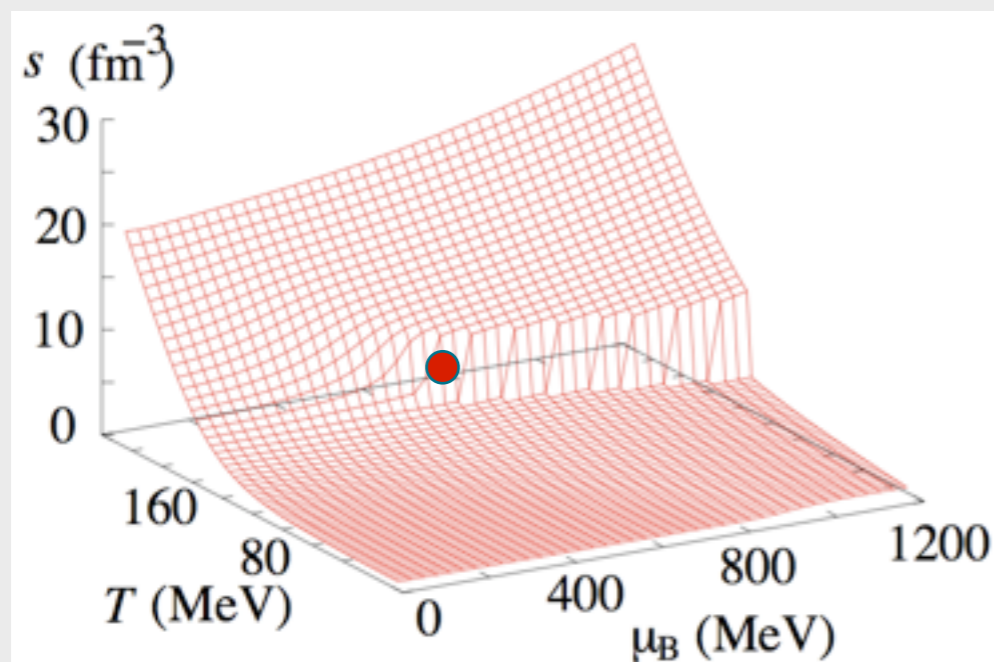
- address all data in the soft sector with one consistent calculation



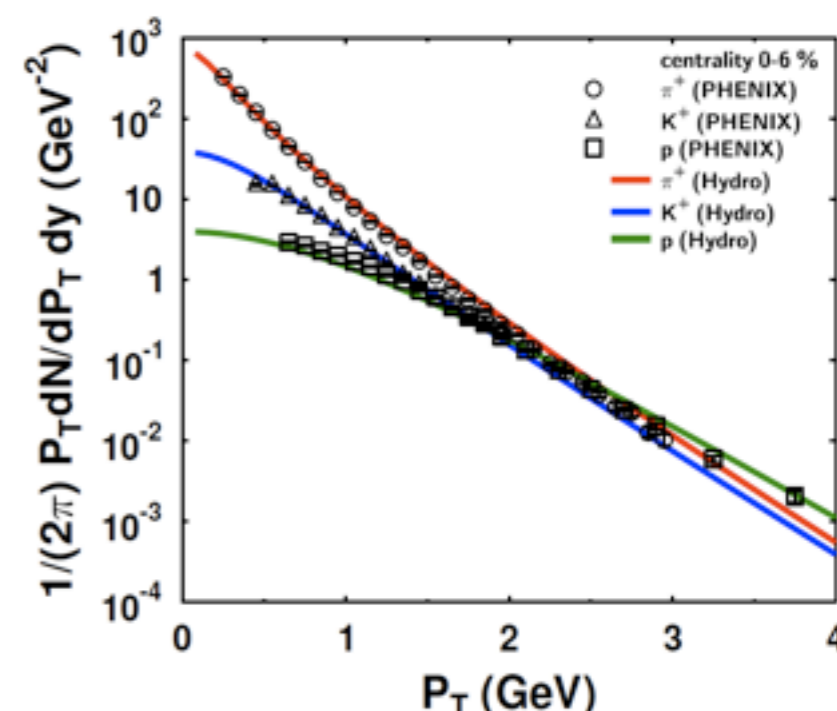
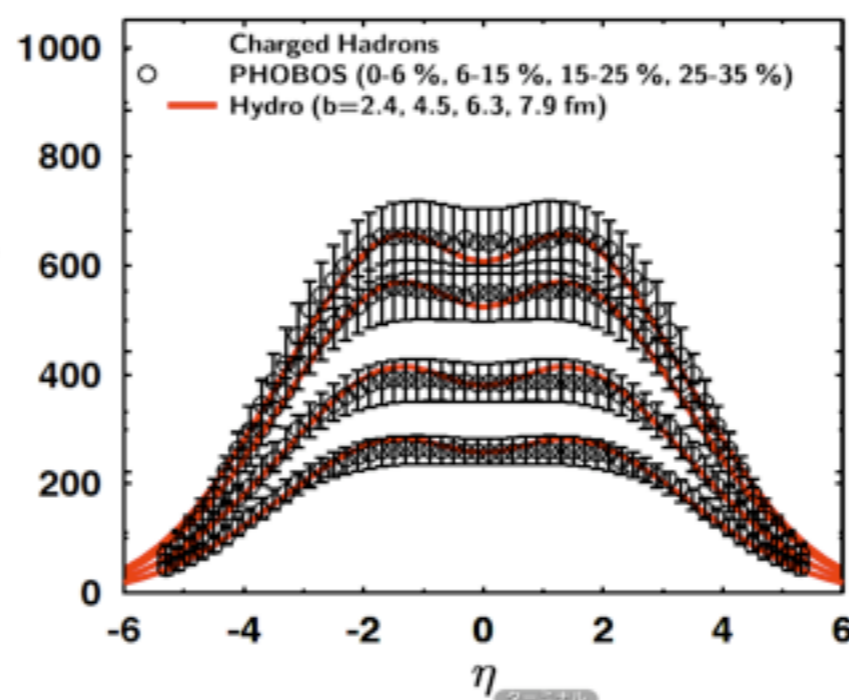
Nonaka & Bass: PRC75, 014902 (2007)
See also Hirano; Kodama et al.

Improved Equation of State

- use EoS with QCD critical point ($T_E=159$ MeV, $\mu_E=550$ MeV)
- interpolate to Lattice parametrization for $\mu_B=0$

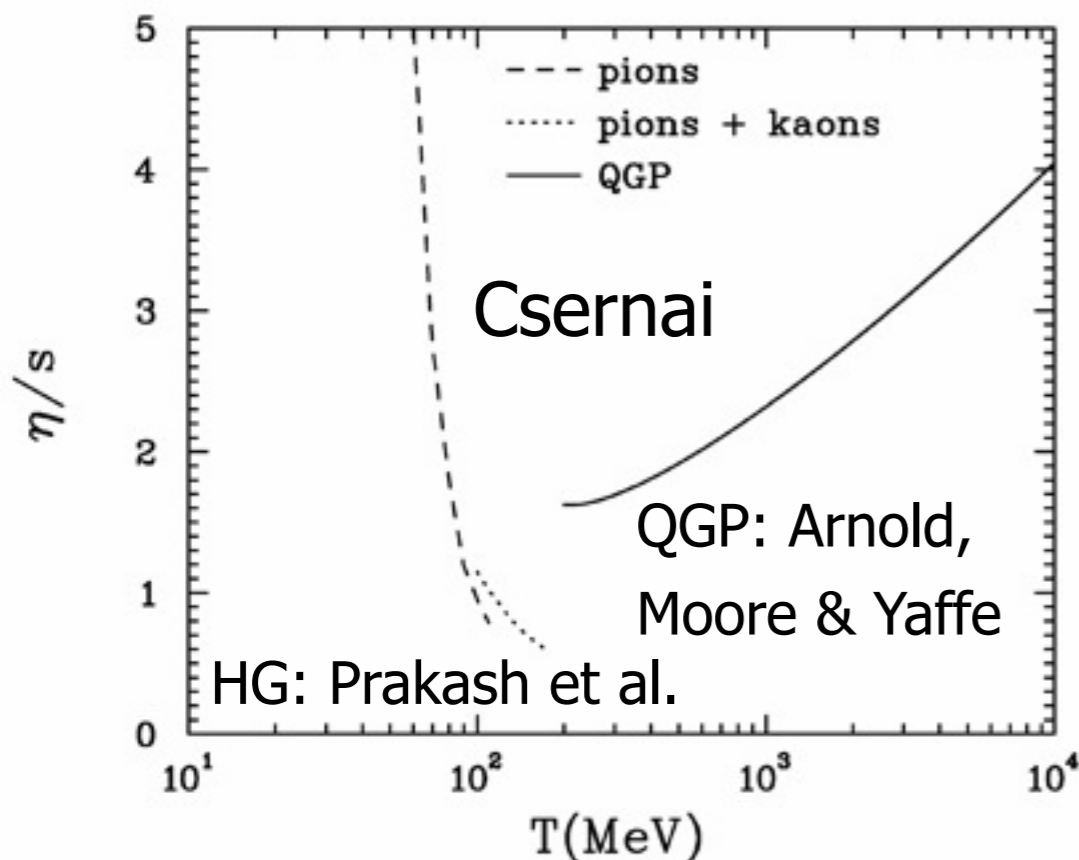
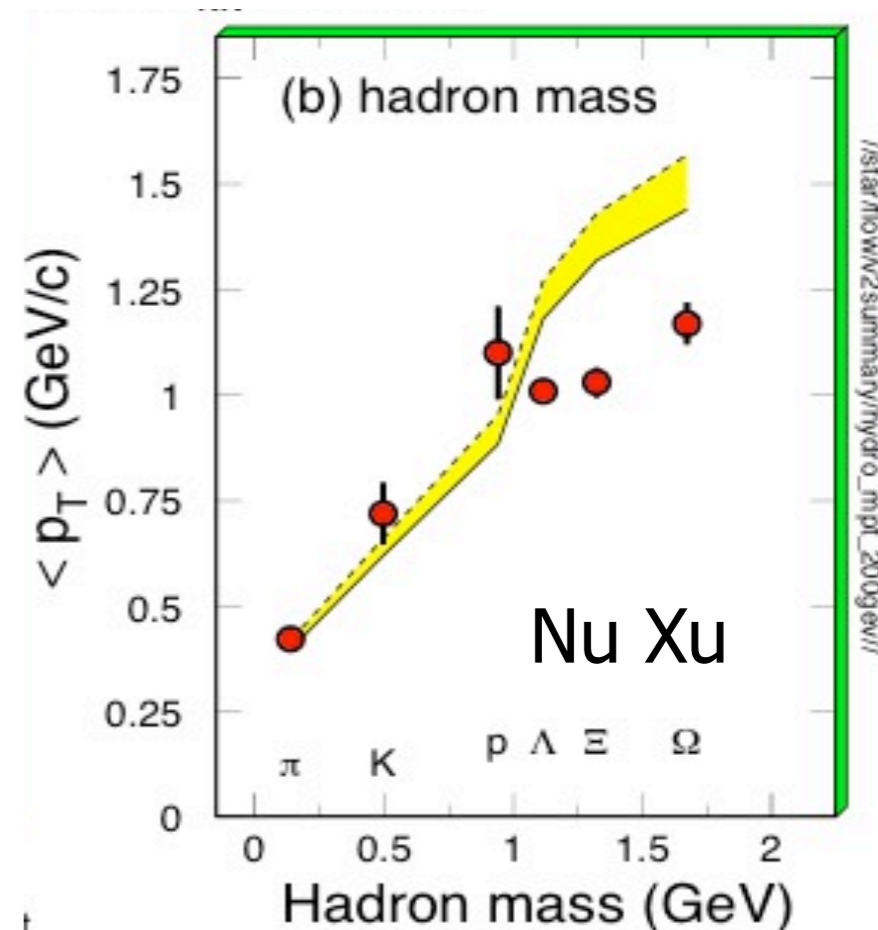


- retune initial conditions to $dN_{ch}/d\eta$ and $1/p_t dN/dp_t$
- now study effect of EoS on reaction dynamics, jet energy-loss and HBT...
- next item to-do: improved initial conditions based on CGC w/ fluctuations



Ideal RFD: Challenges

- centrality systematics of v_2 less than perfect
- no flavor dependence of cross-sections
- separation chemical and kinetic freeze-out:
 - normalize spectra by hand
 - PCE: proper normalization, wrong v_2
- ▶ off-equilibrium effect!



Viscosity:

- success of ideal RFD argues for a low viscosity in QGP phase
- compatible with AdS/CFT bound of $1/4\pi$
- viscosity will strongly change as function of temperature during collision
- need to account for viscous corrections, in particular in the hadronic phase

3D-Hydro + Micro Model

full 3-d ideal RFD

QGP evolution

Hadronization

Cooper-Frye
formula

Monte Carlo

UrQMD

hadronic
rescattering

T_c

T_{sw}

t fm/c

Hydrodynamics

- ideally suited for dense systems
 - model early QGP reaction stage
- well defined Equation of State
- parameters:
 - initial conditions
 - Equation of State

+

micro. transport (UrQMD)

- no equilibrium assumptions
 - model break-up stage
 - calculate freeze-out
 - includes viscosity in hadronic phase
- parameters:
 - (total/partial) cross sections

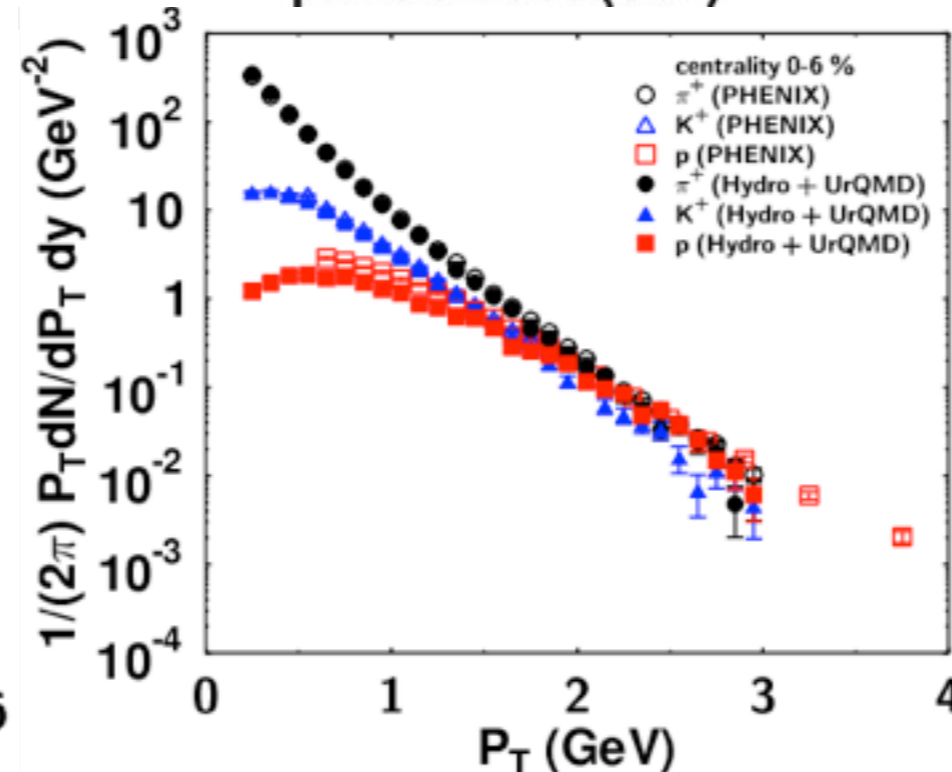
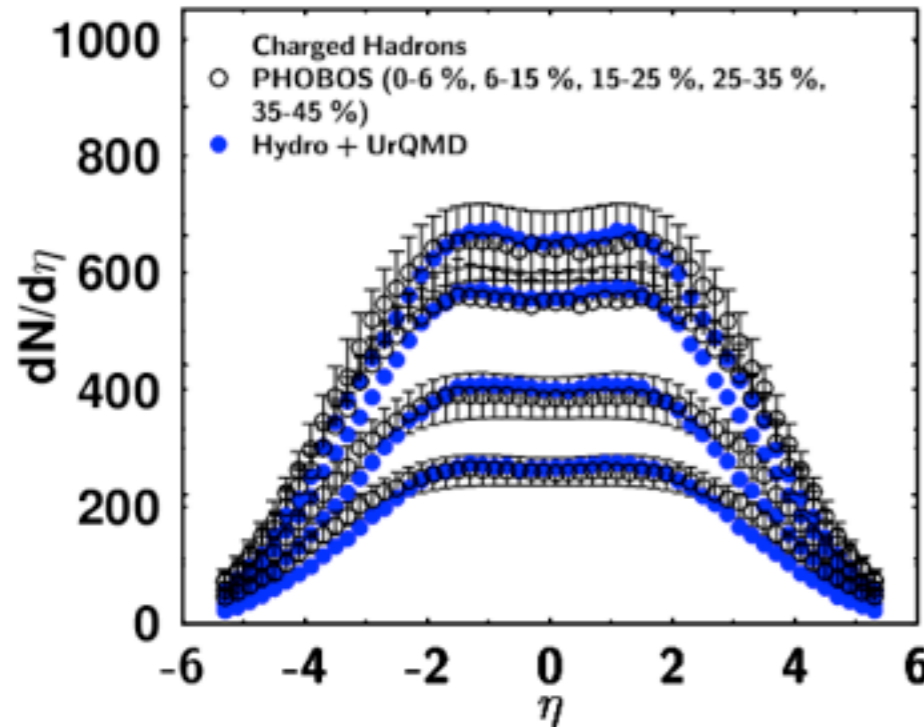
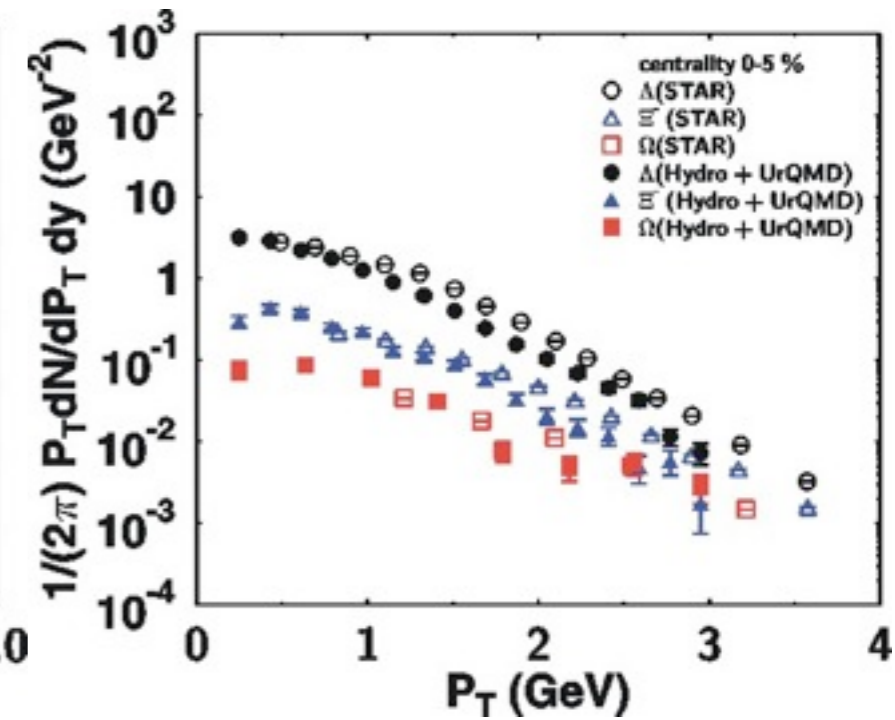
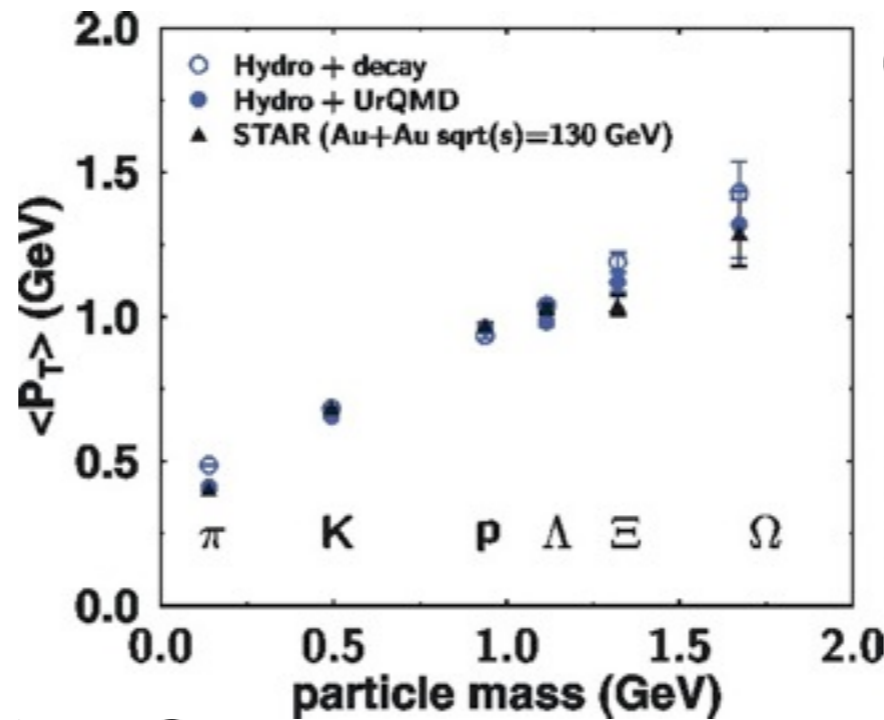
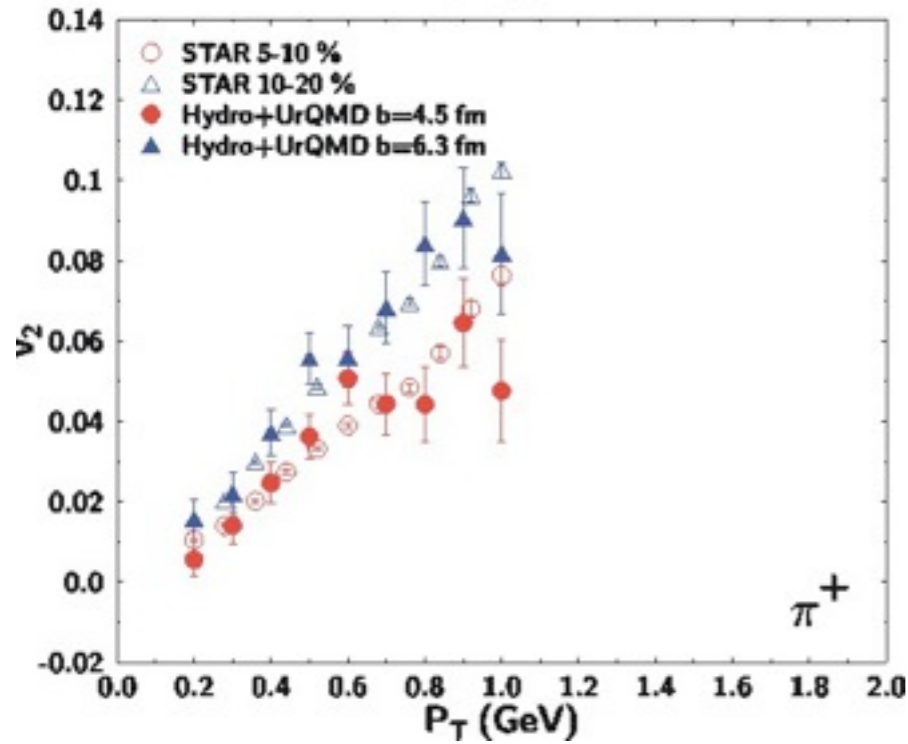
matching condition:

- use same set of hadronic states for EoS as in UrQMD
- generate hadrons in each cell using local T and μ_B

S.A. Bass & A. Dumitru, Phys. Rev. **C61** (2000) 064909
 D. Teaney et al, nucl-th/0110037
 T. Hirano et al. Phys. Lett. **B636** (2006) 299
 C. Nonaka & S.A. Bass, Phys. Rev. **C75** (2006) 014902

3D-Hydro+UrQMD: Results

Au+Au, sqrt(s)=200 GeV



► good description of cross section dependent features & non-equilibrium features of hadronic phase

3D-Hydro+Micro: Outlook

- benchmark comparison between different ideal RFD+Micro implementations (TECHQM project / Nonaka to coordinate)
- study sensitivity to EoS
- explore different initial conditions (Glauber vs. CGC - beware of extreme scenarios!)
- redo comparison to RHIC data with new EoS & initial conditions
- develop Hydro+Micro converters for vRFD
- develop 3+1D vRFD+Micro as standard model for bulk evolution



η/s of a Hadron Gas



Viscosity at RHIC

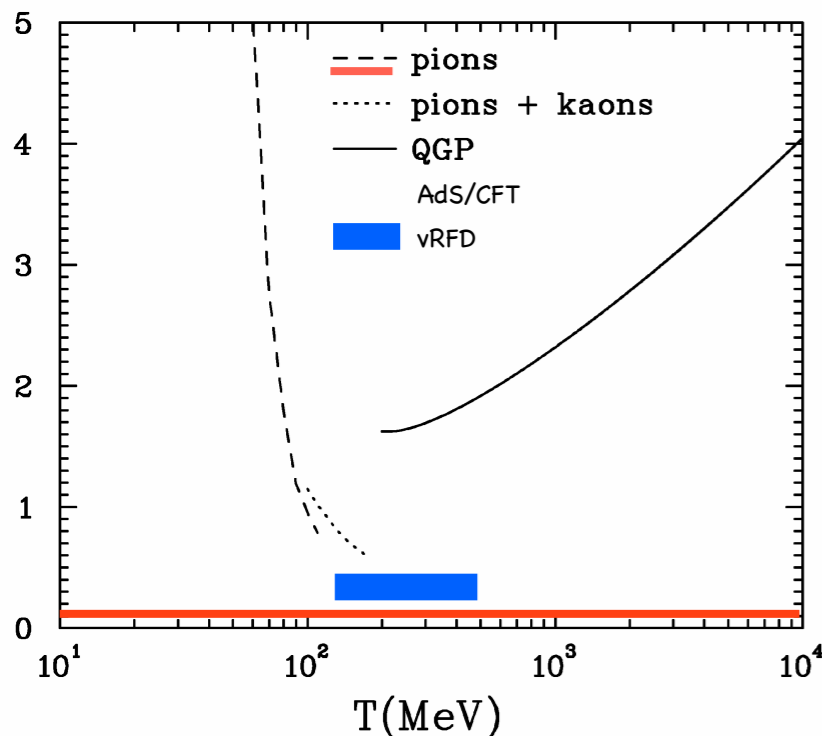
initial state

QGP and
hydrodynamic expansion

hadronic phase
and freeze-out

pre-equilibrium

hadronization



large elliptic flow
& success of ideal RFD:
very small viscosity

expanding hadron gas
w/ significant & increasing
mean free path:
large viscosity

- viscosity of matter @ RHIC changes strongly with time & phase
- how can we learn more about the viscosity of QCD matter?

L.P. Csernai, J.I. Kapusta & L. McLerran: Phys. Rev. Lett. **97**: 152303 (2006)
M. Prakash, M. Prakash, R. Venugopalan & G. Welke: Phys. Rept. **227**, 321 (1993)
P. Arnold, G.D. Moore & L.D. Yaffe: JHEP **05**: 051 (2003)

Microscopic Transport: η/s of a Hadron Gas

- for particles in a fixed volume, the stress energy tensor discretizes

$$\pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} \frac{p^x(j)p^y(j)}{p^0(j)}$$

- and the Green-Kubo formula reads:

$$\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0) \pi^{xy}(t) \rangle$$

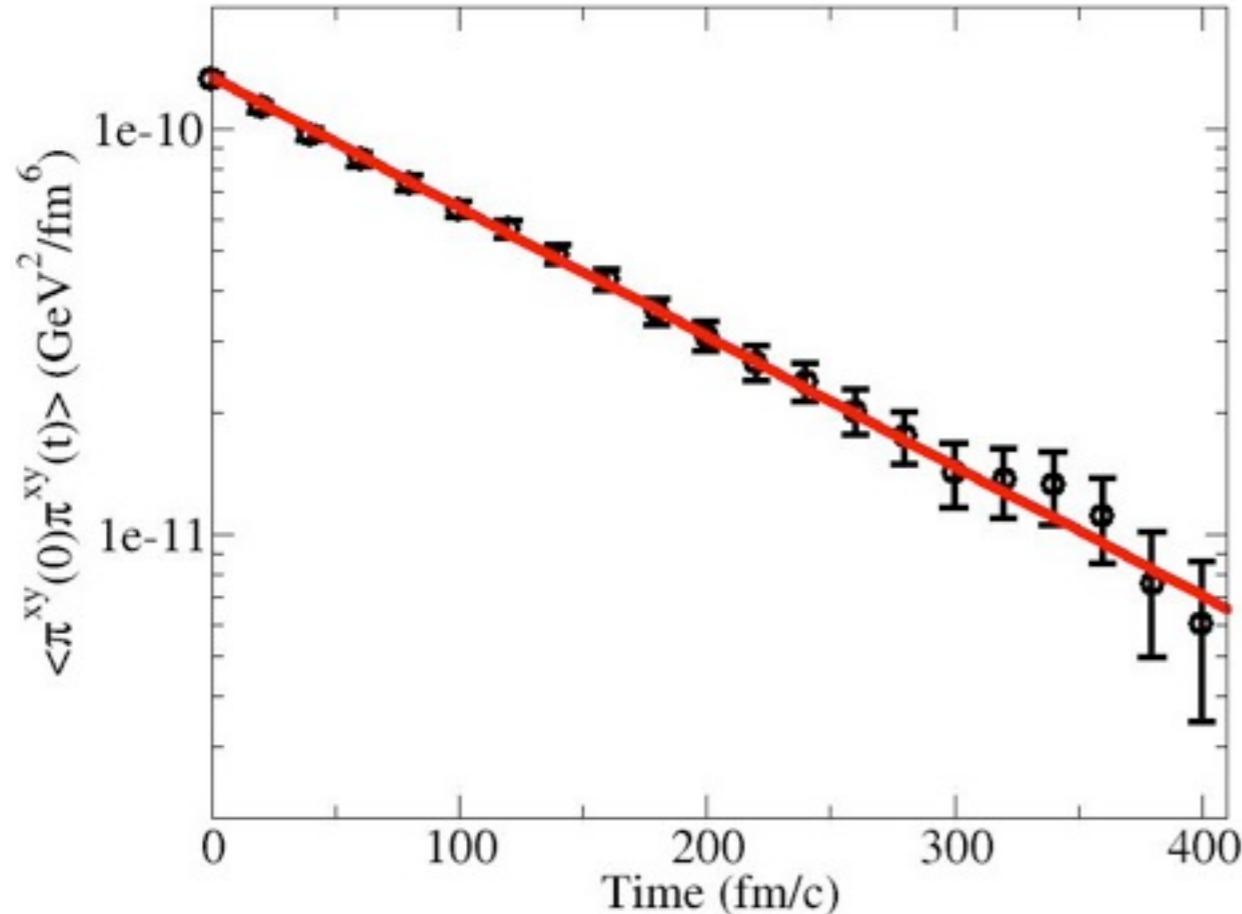
Entropy:

- extract thermodynamic quantities via:

$$P = \frac{1}{3V} \sum_{j=1}^{N_{\text{part}}} \frac{|\vec{p}|^2(j)}{p^0(j)} \quad \epsilon = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} p^0(j)$$

- use Gibbs relation (with chem. pot. extracted via SM)

$$s_{\text{Gibbs}} = \left(\frac{\epsilon + P - \mu_i \rho_i}{T} \right)$$



- evaluating the correlator numerically, e.g. in UrQMD one empirically finds an exponential decay as function of time
- using the following ansatz, one can extract the **relaxation time τ_π** :

$$\langle \pi^{xy}(0) \pi^{xy}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi}\right)$$

- the shear viscosity then can be calculated from known/extracted quantities:

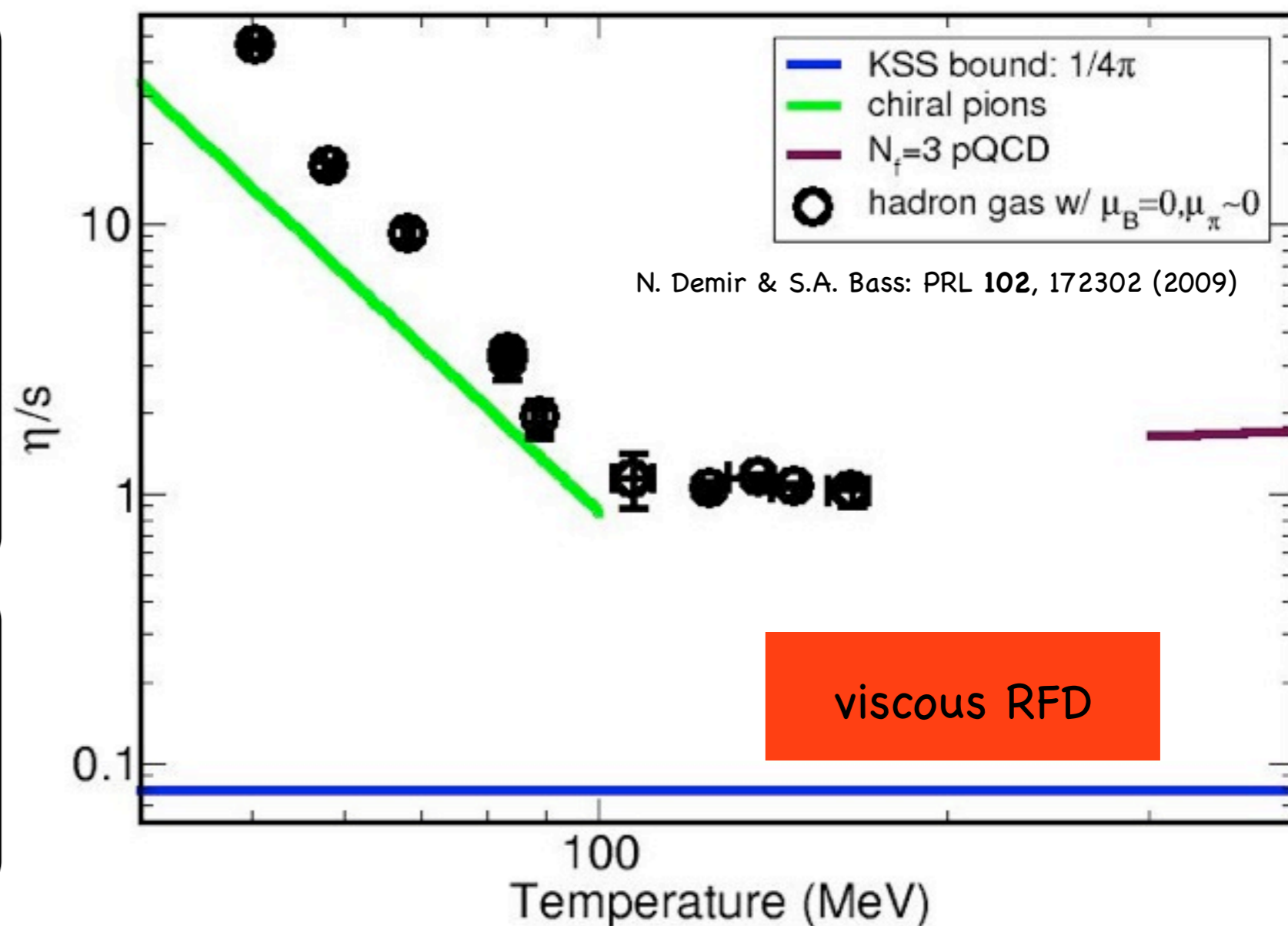
$$\eta = \tau_\pi \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle$$

η/s of a Hadron Gas

first reliable calculation of η/s for a full hadron gas including baryons and anti-baryons

- low temperature trend qualitatively confirms chiral pion calculation
- above $T=100$ MeV: $\eta/s \approx 1$ remains roughly constant
- η/s is a factor of 3-5 above range required by viscous RFD analysis!

- breakdown of vRFD in the hadronic phase?
- what are the consequences for η/s in the deconfined phase?



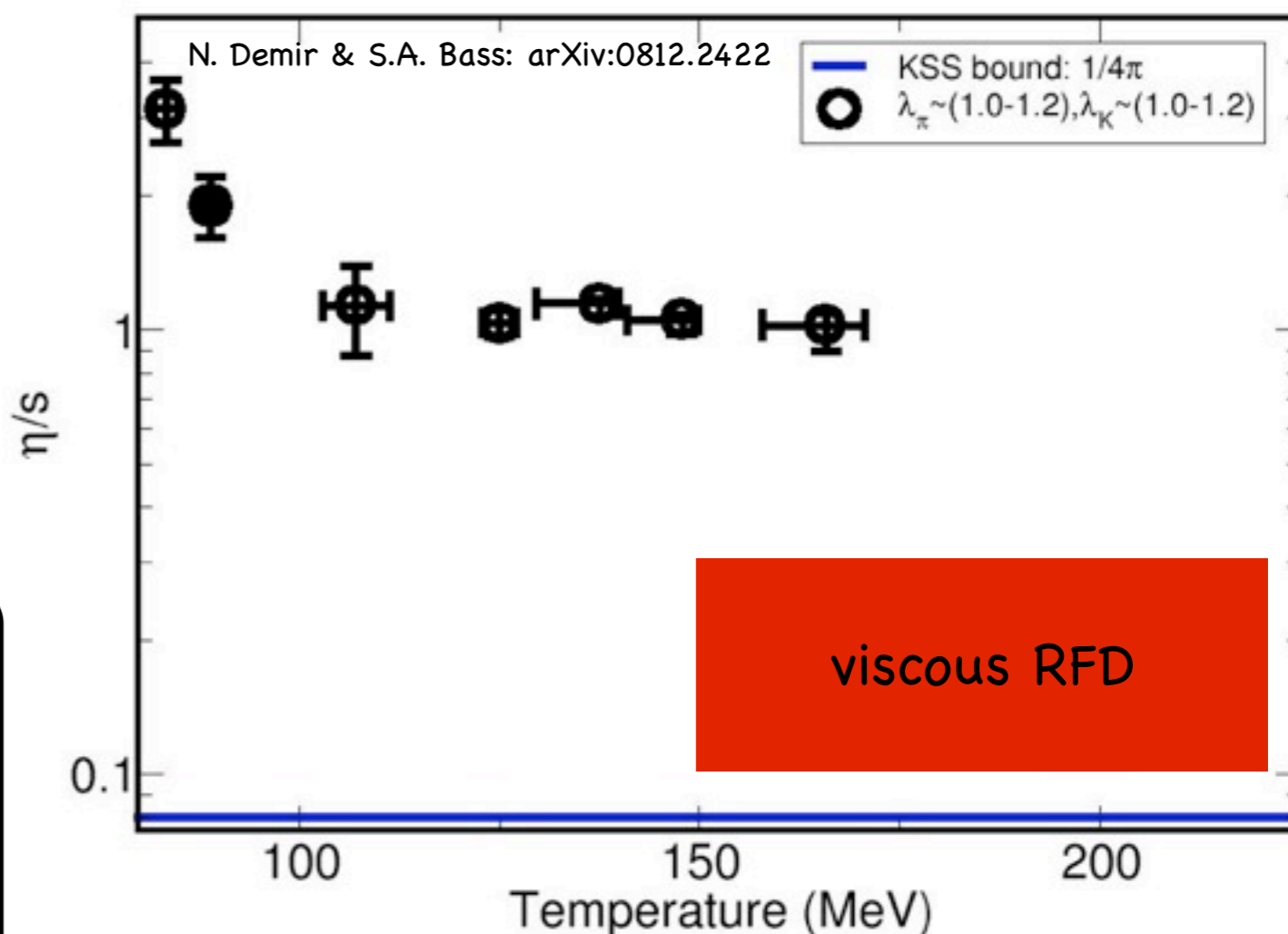
Dynamic Systems: η/s at non-unit fugacities

- freeze-out temperature required by RFD to reproduce spectral shapes: ~ 110 MeV
- temperature extracted from Statistical Model fits to hadron yields/ratios: ~ 160 MeV
- ▶ separation of chemical and kinetic freeze-out in the hadronic phase!
- ▶ picture confirmed by hybrid hydro+micro calculations
- ▶ off-equilibrium effect – implies non-unit species-dependent fugacities in RFD

box calculations w/ non-unit fugacities:

- initialize matter w/ equilibrium distributions, but off-equilibrium yields, corresponding to desired fugacities
- perform viscosity measurement before system relaxes into equilibrium
- verify fugacities at time of measurement w/ statistical model analysis

- non-unit fugacities reduce η/s by a factor of two to $\eta/s \approx 0.5$
- η/s still above value required for viscous RFD fit to data
- ▶ η/s needs to be significantly lower in deconfined phase for vRFD to reproduce elliptic flow data!



T. Hirano & K. Tsuda: Nucl. Phys. **A715**, 821 (2003)
 P.F. Kolb & R. Rapp: Phys. Rev. **C67**, 044903 (2003)

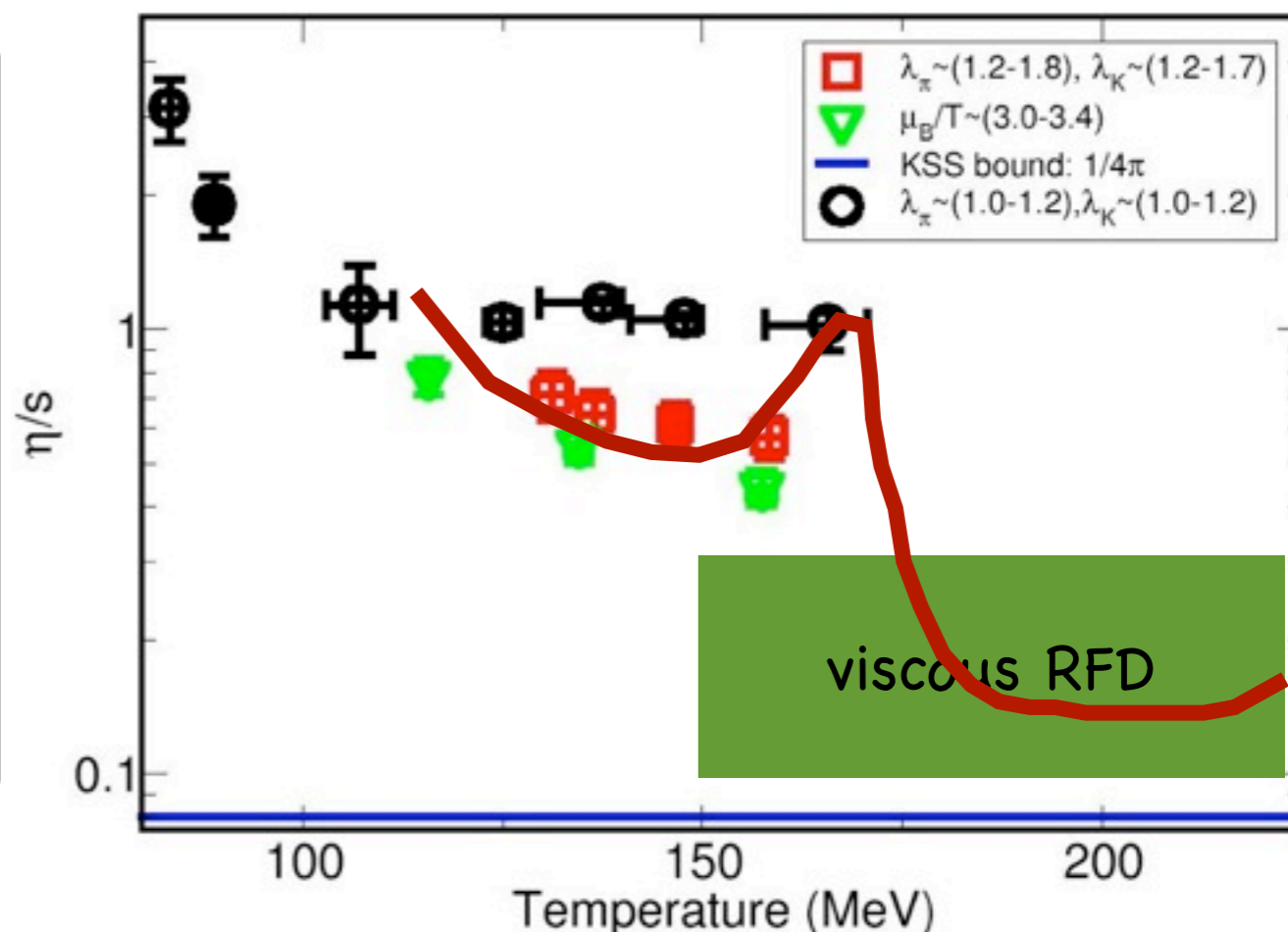
Conclusions and Outlook

Hybrid Hydro+Micro models:

- 3+1D Hydro + Micro models have been very successful in describing the bulk properties of hot & dense QCD matter created at RHIC
- the microscopic treatment of the hadronic phase does not only address viscous effects, but also the inherent off-equilibrium evolution of the system during its break-up stage
- in the future, 3+1D vRFD + Micro models should be pursued, combining the best possible description of the low viscosity deconfined phase with the optimum description of the hadronic phase

Viscosity of QCD matter:

- need to parametrize η/s as function of T for vRFD calculations
- trajectory of η/s in a heavy-ion collision as a function of temperature may have complicated shape
- calculation of hadronic η/s will help to constrain η/s in the deconfined phase

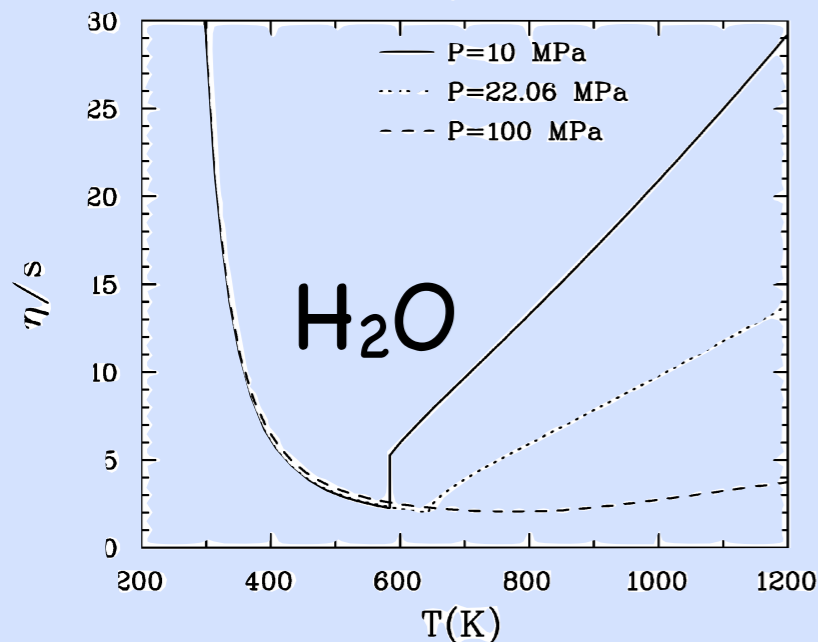
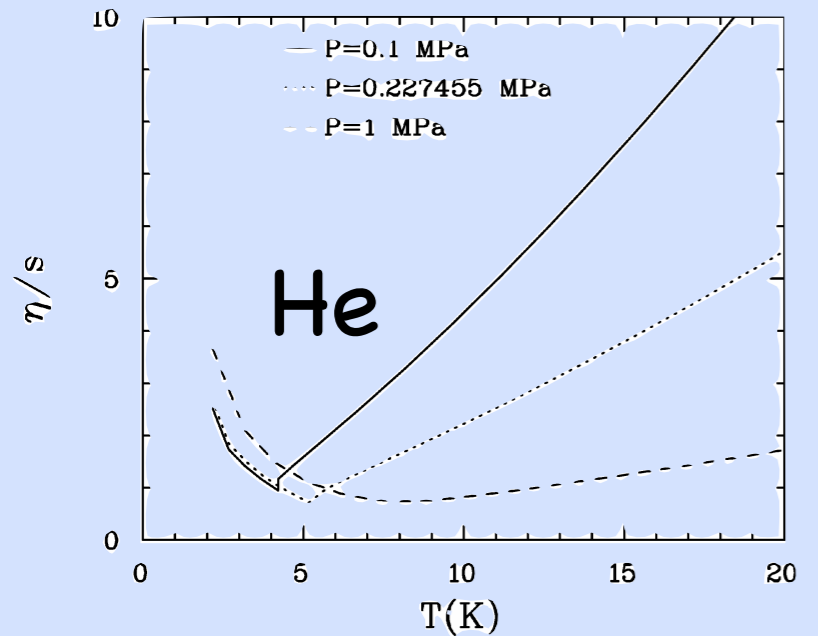




The End

Temperature Dependence of η/s

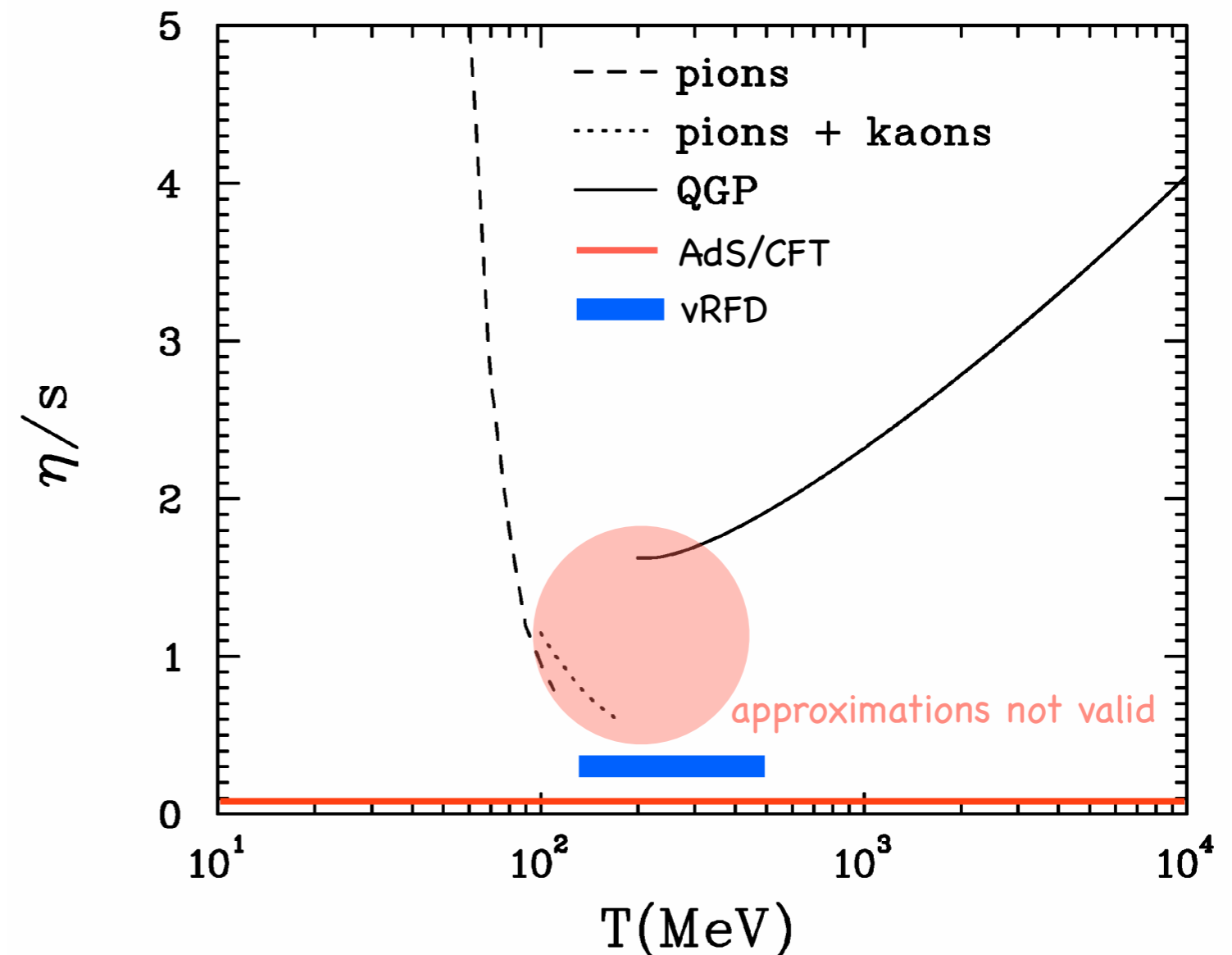
what can ordinary matter, e.g. He or H₂O teach us about η/s ?



- η/s has minimum & discontinuity at T_c

temperature dependence of η/s in QCD can be estimated in low- and high-temperature limit:

- low temperature: chiral pions
- high temperature: QGP in HTL approximation



L.P. Csernai, J.I. Kapusta & L. McLerran: Phys. Rev. Lett. **97**: 152303 (2006)
M. Prakash, M. Prakash, R. Venugopalan & G. Welke: Phys. Rept. **227**, 321 (1993)
P. Arnold, G.D. Moore & L.D. Yaffe: JHEP **05**: 051 (2003)

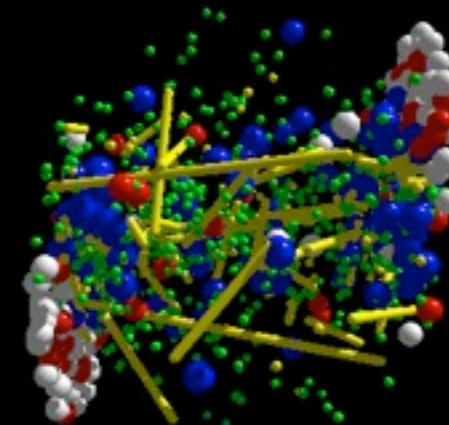
Hadronic Matter: UrQMD

- elementary degrees of freedom: **hadrons**, const. (di)quarks
- classical trajectories in phase-space (relativistic kinematics): evolution of phase-space distribution via Boltzmann Equation:

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \nabla_r \right] f^1 = \mathcal{C}_{\text{coll}}$$

with $\mathcal{C}_{\text{coll}} = N \int \sigma d\Omega \int d\vec{p}_2 |\vec{v}_1 - \vec{v}_2| [f_1(\vec{p}_1') f_1(\vec{p}_2') - f_1(\vec{p}_1) f_1(\vec{p}_2)]$

- initial high energy phase of the reaction is modeled via the excitation and fragmentation of strings
- 55 baryon- and 32 meson species, among those 25 N^* , Δ^* resonances and 29 hyperon/hyperon resonance species
- full baryon-antibaryon and isospin symmetry



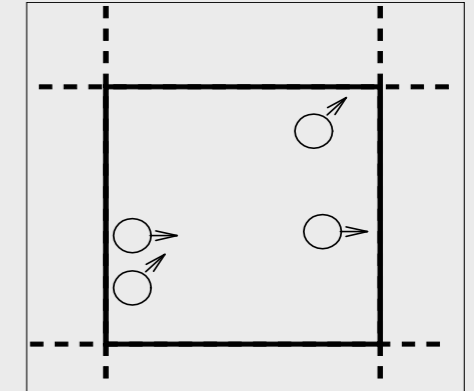
main physics input and parameters:

- cross sections: total and partial cross sections, angular distributions
- resonance parameters: total and partial decay widths
- string fragmentation scheme: fragmentation functions, formation time

Infinite Matter Calculations

Strategy: confine UrQMD to box with periodic boundary conditions

- system will evolve into equilibrium state (no freeze-out occurs)
- need to disable multi-body processes to maintain detailed balance

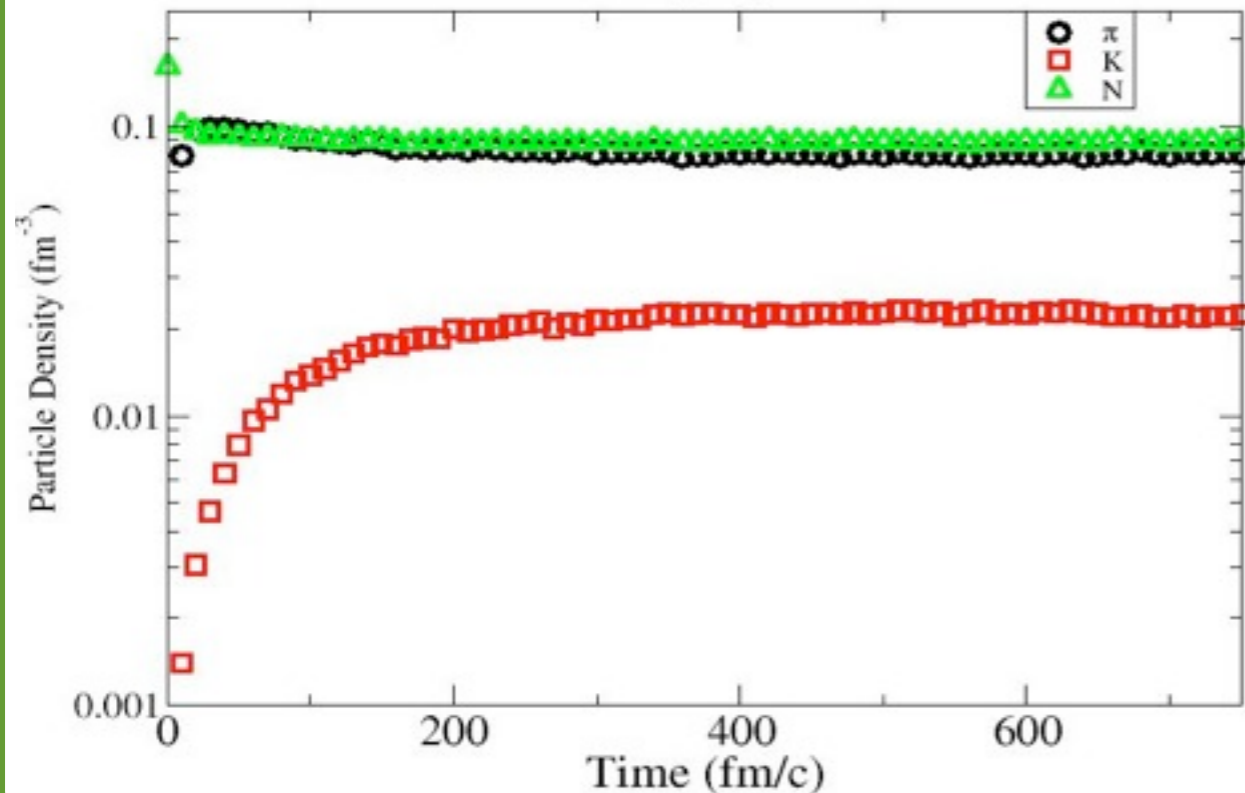


chemical equilibrium:

- particle multiplicities saturate as function of time
- fit to Statistical Model can be used to extract μ_i

$$\varepsilon = 0.3 \text{ (GeV/fm}^3\text{)}$$

$$\rho_B = \rho_0$$

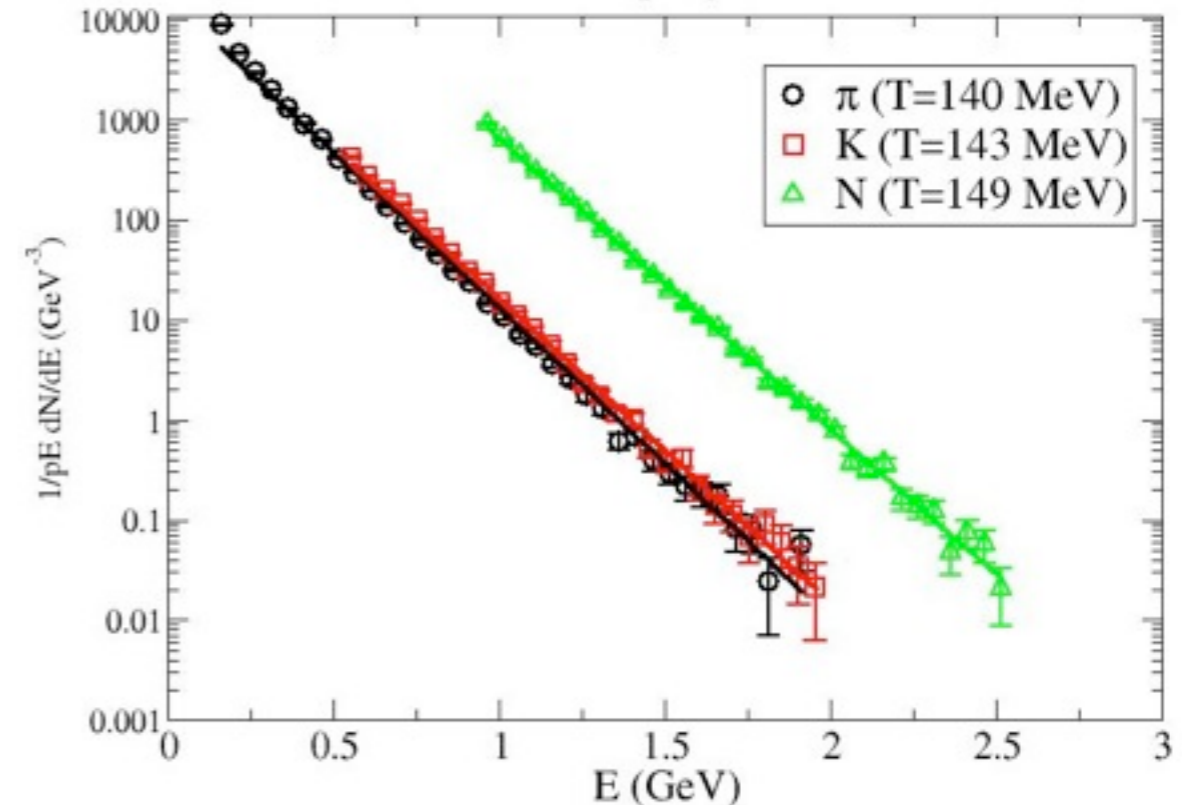


kinetic equilibrium:

- isotropy of momentum distributions
- use energy spectrum to extract temperature

$$\varepsilon = 0.3 \text{ (GeV/fm}^3\text{)}$$

$$\rho_B = \rho_0$$

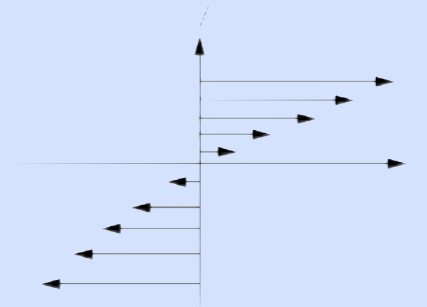


Shear Viscosity: Linear Transport Equation & Green - Kubo Formalism

Mechanical definition of shear viscosity:

- application of a shear force to a system gives rise to a non-zero value of the xy-component of the **pressure tensor** P_{xy} . P_{xy} is then related to the velocity flow field via the **shear viscosity coefficient** η :

$$P_{xy} = -\eta \frac{\partial v_x}{\partial y}$$



- a similar linear transport equation can be defined for other transport coefficients: thermal conductivity, diffusion ...

- using linear-response theory, the **Green-Kubo relations** for the shear viscosity can be derived, expressing η as an integral of an **near-equilibrium time correlation function of the stress-energy tensor**:

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \left\langle \pi^{xy}(\vec{0}, 0) \pi^{xy}(\vec{r}, t) \right\rangle_{\text{equil}}$$

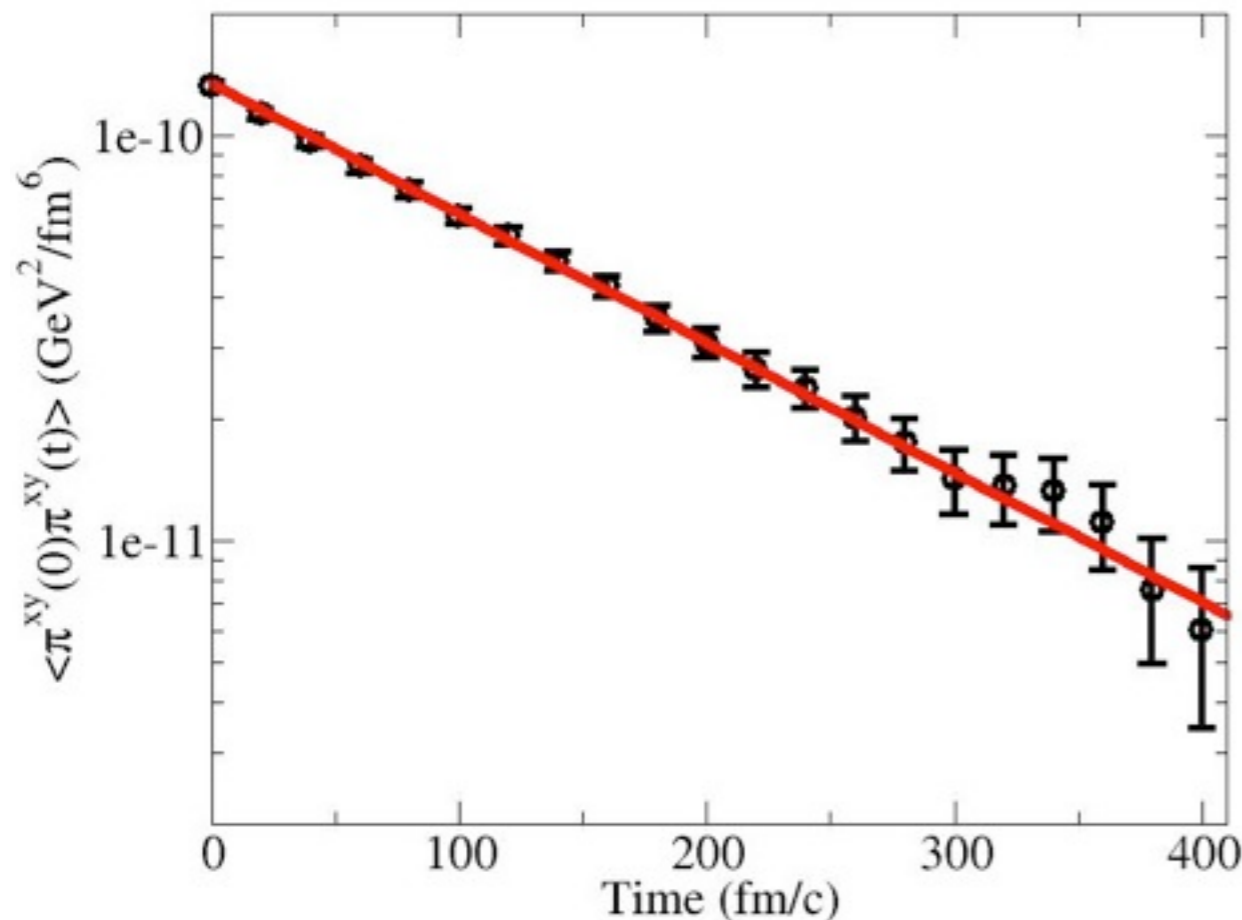
with the stress-energy tensor: $\pi^{\mu\nu}(\vec{r}, t) = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x, p)$

Kubo Formalism in Microscopic Transport

- for a set of discrete particles in a fixed volume, the stress energy tensor discretizes

$$\text{from } \pi^{\mu\nu}(\vec{r}, t) = \int d^3p \frac{p^\mu p^\nu}{p^0} f(x, p) \quad \text{to} \quad \pi^{xy} = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} \frac{p^x(j) p^y(j)}{p^0(j)}$$

- and the Green-Kubo formula reads: $\eta = \frac{V}{T} \int_0^\infty dt \langle \pi^{xy}(0) \pi^{xy}(t) \rangle$



- evaluating the correlator numerically, e.g. in UrQMD one empirically finds an exponential decay as function of time
- using the following ansatz, one can extract the **relaxation time** τ_π :

$$\langle \pi^{xy}(0) \pi^{xy}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi}\right)$$

- the shear viscosity then can be calculated from known/extracted quantities:

$$\eta = \tau_\pi \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle$$

Entropy in Microscopic Transport Models

The extraction of entropy from microscopic transport models is non-trivial:

- use two independent methods to ensure accuracy
- thermodynamic quantities which can be extracted directly from box-calculation are: pressure p , energy-density ϵ , particle number N_i , temperature T and volume V

$$P = \frac{1}{3V} \sum_{j=1}^{N_{\text{part}}} \frac{|\vec{p}|^2(j)}{p^0(j)} \quad \epsilon = \frac{1}{V} \sum_{j=1}^{N_{\text{part}}} p^0(j)$$

Method #1: **Gibbs entropy**

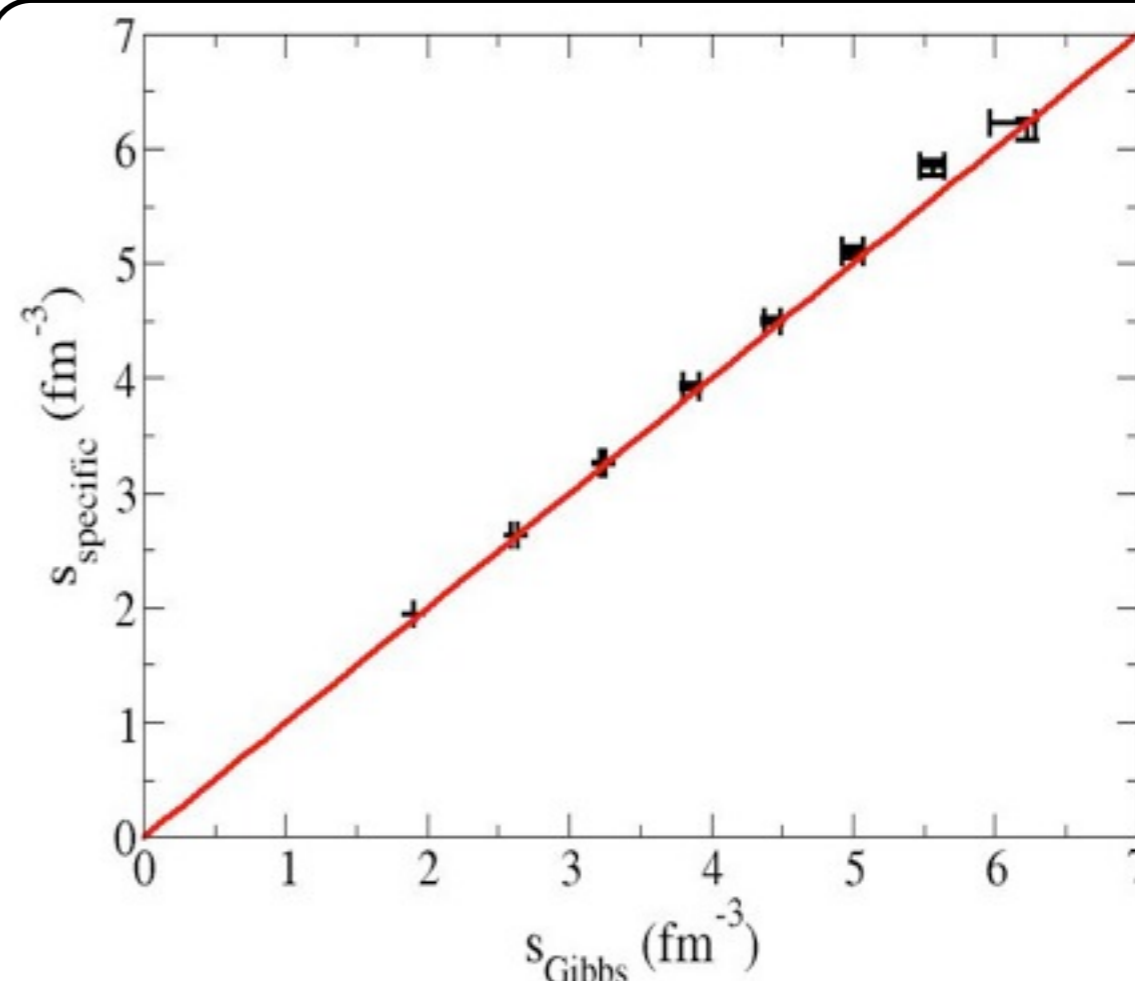
- extract chemical potential(s) from SM
- use Gibbs-relation for entropy:

$$s_{\text{Gibbs}} = \left(\frac{\epsilon + P - \mu_i \rho_i}{T} \right)$$

Method #2: **specific entropy**

- sum over specific entropies of all hadron species, which can be calculated as functions of m/T and μ_B/T :

$$s_{\text{specific}} = \frac{1}{V} \sum_i^{N_{\text{spec}}} \left(\frac{s}{n} \right)_i N_i$$



comparison between Gibbs and specific entropy shows excellent agreement!

Consistency Check: Entropy Scaling

The consistency of the entropy extraction can be verified via a scaling law with the speed of sound of the system:

$$s \sim T \frac{1}{c_s^2}$$

Step #1: determine speed of sound c_s , using pressure and energy-density:

$$c_s^2 = \left(\frac{\partial P}{\partial \epsilon} \right)$$

- analysis yields $c_s^2=0.18$

Step #2: plot Gibbs entropy vs. temperature, using the scaling law

► scaling law is well reproduced

